BSEH Practice Paper (March 2024)

(2023-24)

Marking Scheme

Model Question Paper **SET-A**

MATHEMATICS **CODE: 835**

⇨ Important Instructions: ● All answers provided in the Marking scheme are SUGGESTIVE
● Examiners are requested to accept all possible alternative correct answer(s).

	SECTION – A (1Mark × 20Q)	
Q. No.	**EXPECTED ANSWERS**	**Marks**
Question 1.	Let R be the relation in the set **N** given by R = {(a, b) : a = b -2, b > 6}. Choose the correct answer.	
Solution:	**(C) (6, 8) ∈ R**	1
Question 2.	$\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$ is equal to:	
Solution:	**(B)** $\boldsymbol{\frac{\pi}{6}}$	1
Question 3.	If A = $\begin{bmatrix} \tan\theta & \cot\theta \\ -\cot\theta & \tan\theta \end{bmatrix}$, $0 < \theta < \frac{\pi}{2}$ and $A + A' = 2I$, then the value of θ is:	
Solution:	**(A)** $\boldsymbol{\frac{\pi}{4}}$	1
Question 4.	If a matrix A is both symmetric and skew symmetric, then	
Solution:	**(B) A is a zero matrix**	1
Question 5.	If the vertices of a triangle are (1, 0), (6, 0) and (4, 3), then by using determinants its area is	
Solution:	**(C)** $\boldsymbol{\frac{15}{2}}$	1
Question 6.	If $y = x.\log x$, then $\frac{d^2y}{dx^2}$ is equal to:	
Solution:	**(A)** $\boldsymbol{\frac{1}{x}}$	1
Question 7.	The antiderivative of $\left(\sqrt{x}+\frac{1}{\sqrt{x}}\right)$ equals:	
Solution:	**(C)** $\boldsymbol{\frac{2}{3}x^{\frac{3}{2}}+2x^{\frac{1}{2}}+C}$	1
Question 8.	$\int e^x\left(\frac{1}{x}-\frac{1}{x^2}\right)dx$ equals:	
Solution:	**(B)** $\boldsymbol{\frac{1}{x}e^x+C}$	1
Question 9.	The value of $\int_{-1}^{1} x^5\,dx$ is	
Solution:	(C) 0	1
Question 10.	The order of the differential equation $2x^2\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + y = 0$ is :	
Solution:	(A) 2	1
Question 11.	Which substitution can solve a homogeneous differential equation of the form $\frac{dx}{dy} = h\left(\frac{x}{y}\right)$?	
Solution:	**Put x = vy**	1
Question 12.	The function $f(x) = \begin{cases} \sin x - \cos x & ,\text{if } x \neq 0 \\ k & ,\text{if } x = 0 \end{cases}$ is continuous at x = 0, then find the value of k.	
Solution:	$\lim_{x\to 0} f(x) = \lim_{x\to 0}(\sin x - \cos x)$ $= 0 - 1$ $= -1$ Since f(x) is continuous at x = 0 $\therefore \lim_{x\to 0} f(x) = f(0)$ ⇨ $\boldsymbol{-1 = k}$	1
Question 13.	If a line has the direction ratios 2, -1, -2, then what are its direction cosines?	
Solution:	$\frac{2}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-1}{\sqrt{2^2+(-1)^2+(-2)^2}}, \frac{-2}{\sqrt{2^2+(-1)^2+(-2)^2}}$	1

	⇨ $\frac{2}{3}, \frac{-1}{3}, \frac{-2}{3}$	
Question 14.	Compute P(A|B), if P(B) = 0.5 , P(A∩B) = 0.32 .	
Solution:	$P(A\|B) = \frac{P(A\cap B)}{P(B)}$ $= \frac{0.5}{0.32}$ **$P(A\|B) = \frac{25}{16}$**	1
Question 15.	Two collinear vectors are always equal in magnitude. (True / False)	
Solution:	**False**	1
Question 16.	Two events will be independent, if P(A'B') = [1 – P(A)][1 – P(B)]. (True / False)	
Solution:	**True**	1
Question 17.	The probability of obtaining an even prime number on each die, when a pair of dice is rolled is.__________.	
Solution:	**1/6**	1
Question 18.	If $\vec{a} = 2\hat{\imath} + \hat{\jmath} + 3\hat{k}$ and $\vec{b} = 3\hat{\imath} + 5\hat{\jmath} - 2\hat{k}$, then $\|\vec{a} \times \vec{b}\|$ = ________ **.**	
Solution:	$\sqrt{507}$	
Question 19.	**Assertion (A):** If R is the relation defined in set {1, 2, 3, 4, 5, 6} as R = { (a, b) : b = a +1 } then R is not an equivalence relation. **Reason (R):** A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive.	
Solution:	**(A).** Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)	1
Question 20.	**Assertion (A):** The lines are $\vec{r} = \vec{a_1} + \lambda\,\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\,\vec{b_2}$ are perpendicular, when $\vec{b_1}.\vec{b_2} = 0$. **Reason (R):** The angle θ between the lines $\vec{r} = \vec{a_1} + \lambda\,\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ is given by $\cos\theta = \frac{\vec{b_1}.\vec{b_2}}{\|\vec{b_1}\|.\|\vec{b_2}\|}$.	
Solution:	(A) . Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of the Assertion (A)	1
	SECTION – B (2Marks × 5Q)	
Question 21.	Let L be the set of all lines in a plane and R be the relation in L defined as R = {(L_1, L_2): L_1 is perpendicular to L_2}. Show that R is symmetric but neither reflexive nor transitive.	
Solution:	R is not reflexive, as a line L_1 can't be perpendicular to itself, i.e., $(L_1, L_1) \notin R$. R is symmetric as $(L_1, L_2) \in R$ L_1 is perpendicular to L_2 ⇒ L_2 is perpendicular to L_1 ⇒ $(L_2, L_1) \in R$. $\forall L_1, L_2 \in L$ R is not transitive. Indeed, if L_1 is perpendicular to L_2 and L_2 is perpendicular to L_3, then L_1 can never be perpendicular to L_3. In fact, L_1 is parallel to L_3 i.e., $(L_1, L_2) \in R$, and $(L_2, L_3) \in R$ but $(L_1, L_3) \notin R$.	$\frac{1}{2}$ $\frac{1}{2}$ 1
OR Question 21.	Find the value of: $\cos^{-1}(\frac{1}{2}) + 2\sin^{-1}(\frac{1}{2})$	
Solution:	Let $\cos^{-1}(\frac{1}{2}) = x$. Then $\cos x = 1/2 = \cos(\pi/3)$ $\cos^{-1}(\frac{1}{2}) = \pi/3$ Let $\sin^{-1}(\frac{1}{2}) = y$. Then, $\sin y = 1/2 = \sin(\pi/6)$ $\sin^{-1}(\frac{1}{2}) = \pi/6$	$\frac{1}{2}$ $\frac{1}{2}$

	Now $\cos^{-1}(1/2) + 2\sin^{-1}(1/2) = \pi/3 + (2\pi)/6$ $= \pi/3 + \pi/3$ $= (2\pi)/3$	1
Question 22.	Find the value of a, b, c, and d from the equations: $\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$	
Solution:	Equate the corresponding elements of the matrices: a - b = -1 ...(1) 2a + c = 5 ...(2) 2a - b = 0 ...(3) 3c + d = 13 ...(4) Equation (1) -Equation (3) - a = - 1 ⇒ a=1 Equation (1) ⇒ 1 - b = - 1 ⇒ b = 2 Equation (2) ⇒ 2(1) + c = 5 ⇒ c = 3 Equation (4) ⇒ 3(3) + d = 13 ⇒ d = 4 Therefore, a = 1, b = 2, c = 3 and d = 4	$\frac{1}{2}$ $\frac{1}{2}$ 1
Question 23.	Find the value of *k* so that the function is continuous at the indicated point *f(x)* = $\begin{cases} kx+1, & x \le 5 \\ 3x-5, & x > 5 \end{cases}$ at x = 5.	
Solution:	Given function is *f(x)* = $\begin{cases} kx+1, & x \le 5 \\ 3x-5, & x > 5 \end{cases}$ When x < 5, f(x) = kx + 1: A polynomial is continuous at each point x < 5 When x > 5, f(x)= 3x-5 : A polynomial is continuous at each point x > 5 Now f(5) = 5k + 1 $\lim_{x\to 5} f(x) = \lim_{h\to 0} f(5+h) = 3(5+h) - 5 = 15 + 3h - 5$ $= \lim_{h\to 0}(10 + 3h) = 10 + 3(0) = 10$ ….(1) $\lim_{x\to 5} f(x) = \lim_{h\to 0} f(5-h) = k(5-h) + 1$ $= \lim_{h\to 0} (5k - hk + 1) = 5k + 1$ ….(2) Since function is continuous, therefore, both the equations are equal, Equate both the equations and find the value of k, 10 = 5k + 1 5k = 9 k = 9/5	$\frac{1}{2}$ 1 $\frac{1}{2}$
Question 24.	Verify that the function y = x sin 3x, is a solution of the differential equation $\frac{d^2y}{dx^2} + 9y - 6\cos 3x = 0$	
Solution:	Given: y = x sin 3x Diff. w.r.t. 'x' , and we get	

	$\frac{dy}{dx} = \sin 3x + 3x \cos 3x$ …(1) Again differentiate (1) w.r.t. 'x', we get $\frac{d^2y}{dx^2} = 3 \cos 3x + 3 [\cos 3x + x (- \sin 3x). 3]$ On simplifying the above equation, we get $\frac{d^2y}{dx^2} = 6 \cos 3x - 9x \sin 3x$ …(2) Now, substitute (1) and (2) in the given differential equation, and we get the following: L.H.S = $\frac{d^2y}{dx^2} + 9y - 6 \cos 3x$ $= (6 \cos 3x - 9x \sin 3x) + 9(x \sin 3x) - 6 \cos 3x$ $= 6 \cos 3x - 9x \sin 3x + 9x \sin 3x - 6 \cos 3x$ $= 0 =$ R.H.S As L.H.S = R.H.S, the given function is the solution of the corresponding differential equation.	$\frac{1}{2}$ $\frac{1}{2}$ 1
OR Question 24.	Find the general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$	
Solution:	Since $1+y^2 \neq 0$, therefore separating the variables, the given differential equation can be written as $\frac{dy}{1+y^2} = \frac{dy}{1+x^2}$ …..(1) Integrating both sides of equation (1), we get $\int \frac{dy}{1+y^2} = \int \frac{dy}{1+x^2}$ $\tan^{-1}y = \tan^{-1}x + C$ which is the general solution of equation (1)	$\frac{1}{2}$ $1\frac{1}{2}$
Question 25.	Two balls are drawn at random with replacement from a box containing 10 black and 8 red balls. Find the probability that both balls are red.	
Solution:	Total number of balls = 10 black balls + 8 red balls = 18 balls Probability of getting a red ball in the first draw = $\frac{8}{18} = \frac{4}{9}$ As the ball is replaced after the first throw, $\therefore$ Probability of getting a red ball in the second draw = $\frac{8}{18} = \frac{4}{9}$ Since the two balls are drawn with replacement, the two draws are independent. P(both balls are red) = P(first ball is red) x P(second ball is red) Now, the probability of getting both balls red = $\frac{4}{9} \times \frac{4}{9} = \frac{16}{81}$	$\frac{1}{2}$ $\frac{1}{2}$ 1

	SECTION – C (3Marks × 8Q)	
Question 26.	Let A = **R** – {3} and **B =** R – {1}. Consider the function f : A → B defined by $f(x) = \left(\frac{x-2}{x-3}\right)$. Is f one one and onto? Justify your answer.	
Solution:	A = R - {3} and B = R - {1} f : A → B defined by f(x) = (x - 2) / (x - 3) Let (x, y) ∈A then $f(x) = \frac{(x-2)}{(x-3)}$ and $f(y) = \frac{(y-2)}{(y-3)}$ For f(x) = f(y) $\frac{(x-2)}{(x-3)} = \frac{(y-2)}{(y-3)}$ (x - 2)(y - 3) = (y - 2)(x - 3) x y - 3x - 2y + 6 = xy - 3y - 2x + 6 - 3x - 2y = - 3y - 2x - 3x + 2x = - 3y + 2y - x = - y x = y Therefore, f is a one-one function. Again, $y = f(x) = \frac{(x-2)}{(x-3)}$ $y = \frac{(x-2)}{(x-3)}$ y(x – 3) = x - 2 xy - 3y = x - 2 x(y – 1) = 3y - 2 or $x = \frac{(3y-2)}{(y-1)}$ Now, $f\left(\frac{3y-2}{y-1}\right) =$ $\Rightarrow \frac{\frac{3y-2}{y-1}-2}{\frac{3y-2}{y-1}-3} = y$ f(x) = y Therefore, f is onto function.	$\frac{1}{2}$ 1 $\frac{1}{2}$ 1
OR Question 26.	$\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$, $\lvert x\rvert < 1$, $y > 0$ and $xy < 1$	
Solution:	Put x = tanθ and y = tanϕ, we have $\tan\frac{1}{2}\left[\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right)\right]$ $= \tan\frac{1}{2}\left[\sin^{-1}\sin 2\theta + \cos^{-1}\cos 2\phi\right]$ $= \tan\frac{1}{2}\left[2\theta + 2\phi\right]$ $= \tan(\theta+\phi)$ $= \frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi}$	$\frac{1}{2}$ $1\frac{1}{2}$ 1

	$= \dfrac{x+y}{1-xy}$	
Question 27.	Find X and Y, if $2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$	
Solution:	$2X + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ ...(1) $3X + 2Y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$ ...(2) Multiply equation (1) by 2, $4X + 6Y = 2\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix}$ ….(3) Multiply equation (2) by 3 $9X + 6Y = 3\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix}$ ….(4)	1
	Subtract equation (4) from (3) $-5X = \begin{bmatrix} 4 & 6 \\ 8 & 0 \end{bmatrix} - \begin{bmatrix} 6 & -6 \\ -3 & 15 \end{bmatrix} = \begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$ $X = -1/5\begin{bmatrix} -2 & 12 \\ 11 & -15 \end{bmatrix}$ $X = \begin{bmatrix} 2/5 & -12/5 \\ -11/5 & 3 \end{bmatrix}$	1
	Substitute this value of X in equation (1) $2\begin{bmatrix} 2/5 & -12/5 \\ -11/5 & 3 \end{bmatrix} + 3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ $3Y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} 4/5 & -24/5 \\ -22/5 & 6 \end{bmatrix}$ $Y = 1/3\begin{bmatrix} 6/5 & 39/5 \\ 42/5 & -6 \end{bmatrix}$ $Y = \begin{bmatrix} 2/5 & -8/5 \\ 14/5 & -2 \end{bmatrix}$	1
Question 28.	Find $\frac{dy}{dx}$ of the function $y^x = x^y$	
Solution:	Given: $y^x = x^y$ $x^y = y^x$ Taking log on both sides $\log(x^y) = \log(y^x)$ $y.\log x = x.\log y$	**1**
	$\frac{d}{dx}(y.\log x) = \frac{d}{dx}(x.\log y)$ $y.\frac{1}{x} + \log x\frac{dy}{dx} = x.\frac{1}{y}\frac{dy}{dx} + \log y.1$	

	$(\log x - \frac{x}{y}) . \frac{dy}{dx} = \log y - \frac{x}{y}$ $\left(\frac{y \log(x) - x}{y}\right) \frac{dy}{dx} = \frac{x \log(y) - y}{x}$ $\frac{dy}{dx} = \frac{y(x \log(y) - y)}{x(y \log(x) - x)}$	$1\frac{1}{2}$ $\frac{1}{2}$
Question 29.	Find the intervals in which the function f is given by $f(x) = 2x^3 - 3x^2 - 36x + 7$ is strictly increasing or strictly decreasing.	
Solution:	Given function: $f(x) = 2x^3 - 3x^2 - 36x + 7$ Diff. w.r.t. 'x' $f'(x) = 6x^2 - 6x + 36 = 6(x^2 - x - 6)$ $f'(x) = 6(x - 3)(x + 2)$,.......(1) Now for increasing or decreasing, $f'(x) = 0$ $6(x - 3)(x + 2) = 0$ $x - 3 = 0$ or $x + 2 = 0$ $x = 3$ or $x = -2$ Therefore, we have sub-intervals are $(-\infty, -2)$, $(-2, 3)$ and $(3, \infty)$ For interval $(-\infty, -2)$, picking $x = -3$, from equation (1), $f'(x) = (+ve)(-ve)(-ve) = (+ve) > 0$ Therefore, f is strictly increasing in $(-\infty, -2)$ For interval $(-2, 3)$, picking $x = 0$, from equation (1), $f'(x) = (+ve)(-ve)(+ve) = (-ve) < 0$ Therefore, f is strictly decreasing in $(-2, 3)$. For interval $(3, \infty)$, picking $x = 4$, from equation (1), $f'(x) = (+ve)(+ve)(+ve) = (+ve) > 0$ Therefore, is strictly decreasing in $(3, \infty)$. So, f is strictly increasing in $(-\infty, -2)$ and $(3, \infty)$. f is strictly decreasing in $(-2, 3)$.	1 $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$ $\frac{1}{2}$
Question 30.	Integrate: $\int x^2 \log x \, dx$	
Solution:	It is given that $I = \int x^2 . \log x \, dx$ Here by taking x as first function and x^2 as second function. Now integrating by parts we get $I = \log x \int x^2 \, dx - \int\{\frac{d}{dx}(\log x) . \int x^2 \, dx\}.dx$ So we get $= \log(x). \frac{x^3}{3} - \int \frac{1}{x}. \frac{x^3}{3} \, dx$ By multiplying the terms $= \frac{x^3.\log x}{3} - \int \frac{x^2}{3} \, dx$ It can be written as $= \frac{x^3.\log x}{3} - \frac{x^3}{9} + C$	$\frac{1}{2}$ 1 $1\frac{1}{2}$
OR Question 30.	Evaluate: $\int_{-5}^{5} \lvert x + 2 \rvert \, dx$	

Solution:	$I = \int_{-5}^{5} \lvert x+2 \rvert \, dx$ We know $\lvert x+2 \rvert = \begin{cases} -(x+2), & x \le -2 \\ (x+2), & x > -2 \end{cases}$ $I = \int_{-5}^{-2} \lvert x+2 \rvert \, dx + \int_{-2}^{5} \lvert x+2 \rvert \, dx$ $I = \int_{-5}^{-2} -(x+2) \, dx + \int_{-2}^{5} (x+2) \, dx$ $I = \left. \frac{-(x+2)^2}{2} \right\rvert_{-5}^{-2} + \left. \frac{(x+2)^2}{2} \right\rvert_{-2}^{5}$ $I = \left(\frac{-(0)^2}{2} - \frac{-(-3)^2}{2} \right) + \left(\frac{(7)^2}{2} - \frac{(0)^2}{2} \right)$ $I = \frac{9}{2} + \frac{49}{2}$ $I = 29$	$\frac{1}{2}$ $1\frac{1}{2}$ 1
Question 31.	The two adjacent sides of a parallelogram are $2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\hat{\imath} - 2\hat{\jmath} - 3\hat{k}$. Find the unit vector parallel to its diagonal.	
Solution:	Adjacent sides of a parallelogram are given as: $\vec{a} = 2\hat{\imath} - 4\hat{\jmath} + 5\hat{k}$ and $\vec{b} = \hat{\imath} - 2\hat{\jmath} - 3\hat{k}$ We know that, the diagonal of a parallelogram is given by $\vec{a} + \vec{b}$ $\vec{a} + \vec{b} = (2+1)\hat{\imath} + (-4-2)\hat{\jmath} + (5-3)\hat{k} = 3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}$ $\lvert \vec{a} + \vec{b} \rvert = \sqrt{(3)^2 + (-6)^2 + (2)^2}$ Hence, the unit vector parallel to the diagonal is $\frac{\vec{a} + \vec{b}}{\lvert \vec{a} + \vec{b} \rvert} = \frac{3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{\sqrt{(3)^2 + (-6)^2 + (2)^2}}$ $= \frac{3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{\sqrt{9+36+4}}$ $= \frac{3\hat{\imath} - 6\hat{\jmath} + 2\hat{k}}{7}$ $= \frac{3}{7}\hat{\imath} - \frac{6}{7}\hat{\jmath} + \frac{2}{7}\hat{k}$	1 1 1
	SECTION – C (5Marks × 4Q)	
Question 32.	If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of equations $2x - 3y + 5z = 11$ $3x + 2y - 4z = -5$ $x + y - 2z = -3$	
Solution:	$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$ $\lvert A \rvert = 2(-4+4) + 3(-6+4) + 5(3-2) = 2(0) + 3(-2) + 5(1)$ $= -6 + 5$ $= -1 \neq 0$; Inverse of matrix exists.	1

Find the inverse of matrix:
Cofactors of matrix:

$A_{11} = 0$, $A_{12} = 2$, $A_{13} = 1$

$A_{21} = -1$, $A_{22} = -9$, $A_{23} = -5$

$A_{31} = 2$, $A_{32} = 23$, $A_{33} = 13$

$$\text{adj.A} = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$ $1\frac{1}{2}$

So,

$$A^{-1} = \frac{1}{-1}\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}$$ 1

Now, matrix of equations can be written as: AX=B

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

And, $X = A^{-1} B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Therefore, x = 1, y = 2 and z =3. $1\frac{1}{2}$

Question 33. Find the shortest distance between the lines l_1 and l_2 whose vector equations are $\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$

Solution:

$\vec{r} = \hat{\imath} + \hat{\jmath} + \lambda(2\hat{\imath} - \hat{\jmath} + \hat{k})$ ...(1)

and $\vec{r} = 2\hat{\imath} + \hat{\jmath} - \hat{k} + \mu(3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$ ...(2)

Comparing (1) and (2) with $\vec{r} = \overrightarrow{a_1} + \lambda\, \overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\, \overrightarrow{b_2}$ respectively,

we get

$\overrightarrow{a_1} = \hat{\imath} + \hat{\jmath}$, and $\overrightarrow{b_1} = 2\hat{\imath} - \hat{\jmath} + \hat{k}$

$\overrightarrow{a_2} = 2\hat{\imath} + \hat{\jmath} - \hat{k}$ and $\overrightarrow{b_2} = 3\hat{\imath} - 5\hat{\jmath} + 2\hat{k}$ 1

Therefore

$\overrightarrow{a_2} - \overrightarrow{a_1} = \hat{\imath} - \hat{k}$

and $\frac{1}{2}$

$\overrightarrow{b_1} \times \overrightarrow{b_2} = (2\hat{\imath} - \hat{\jmath} + \hat{k}) \times (3\hat{\imath} - 5\hat{\jmath} + 2\hat{k})$

$= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{\imath} - \hat{\jmath} - 7\hat{k}$ $1\frac{1}{2}$

$|\overrightarrow{b_1} \times \overrightarrow{b_2}| = \sqrt{9 + 1 + 49} = \sqrt{59}$ 1

Hence, the shortest distance between the given lines is given by

$D = \frac{|(\overrightarrow{a_2} - \overrightarrow{a_1}).(\overrightarrow{b_1} \times \overrightarrow{b_2})|}{|\overrightarrow{b_1} \times \overrightarrow{b_2}|} = \frac{|3 - 0 + 7|}{\sqrt{59}} = \frac{10}{\sqrt{59}}$ 1

OR
Question 33.

Find the vector equation of the line passing through the point (1,2,−4) and perpendicular to the two lines :
$\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$

Solution:

The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\,\vec{b}$ .It is given that, the line passes through (1, 2, -4)

So, $\vec{a} = 1\hat{\imath} + 2\hat{\jmath} - 4\hat{k}$

Given lines are $\frac{x-8}{3} = \frac{y+19}{-16} = \frac{z-10}{7}$ and $\frac{x-15}{3} = \frac{y-29}{8} = \frac{z-5}{-5}$ 1

It is also given that, line is perpendicular to both given lines. So we can say that the line is perpendicular to both parallel vectors of two given lines.

We know that, $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ & $\vec{b}$, so let $\vec{b}$ is cross product of parallel vectors of both lines i.e. $\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2}$ 2
where $\overrightarrow{b_1} = 3\hat{\imath} - 16\hat{\jmath} + 7\hat{k}$ and $\overrightarrow{b_2} = 3\hat{\imath} - 8\hat{\jmath} - 5\hat{k}$

and Required Normal

$\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 3 & -16 & 7 \\ 3 & 8 & -5 \end{vmatrix}$

$= \hat{\imath}(80 - 56) - \hat{\jmath}(-15 - 21) + \hat{k}(24 + 48)$ 1

$\vec{b} = 24\hat{\imath} + 36\hat{\jmath} + 72\hat{k}$

Now, by substituting the value of $\vec{a}$ & $\vec{b}$ in the formula $\vec{r} = \vec{a} + \lambda\,\vec{b}$, we get

$\vec{r} = (1\hat{\imath} + 2\hat{\jmath} - 4\hat{k}) + \lambda(24\hat{\imath} + 36\hat{\jmath} + 72\hat{k})$ 1

Question 34.

Find the area of the region bounded by the curve $y^2 = x$ and the lines x = 1, x = 4 and x-axis in the first quadrant.

Solution:

Equation of the curve is $y^2 = x$.
It is a rightward parabola having vertex at origin and symmetrical about x-axis. x = 1 and x = 4 are two straight lines parallel to y-axis.
$y = \sqrt{x}$ ....(1) x = 1 and x = 4

Points of intersections of given curves
At x = 1, $y = \sqrt{1} = \pm 1$ points are (1, 1) (1, -1)
At x = 4, $y = \sqrt{4} = \pm 2$ points are (4, 2) (4, -2)
∴ points in first quadrant A(1, 1) B(4, 2) C(4, 0), D(1, 0) $1\frac{1}{2}$

Make a rough hand sketch of given curves by taking some corresponding values of x and y.

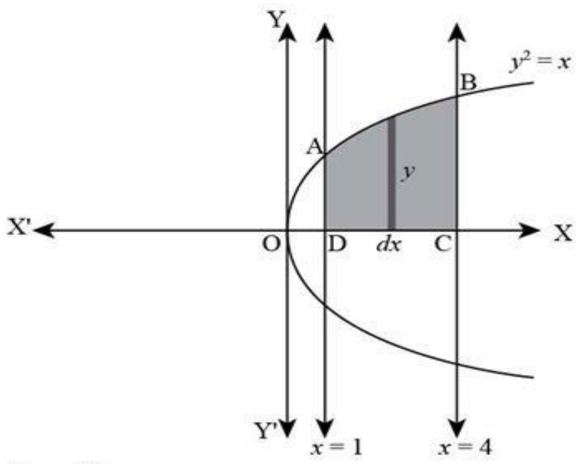


Figure (1)

Required area is shaded region ABCD: **1**

$\left|\int_1^4 y\,dx\right| = \left|\int_1^4 \sqrt{x}\,dx\right|$ [From equation (1)]

$= \left|\int_1^4 x^{1/2}\,dx\right|$

$\left|\frac{x^{3/2}}{3/2}\right|_1^4$ **$1\frac{1}{2}$**

$= \frac{2}{3}\left|(4^{3/2} - 1^{3/2})\right|$

$= \frac{2}{3}\left|(8 - 1)\right| = \frac{2}{3}(7) = \frac{14}{3}$ sq. units **1**

OR
Question 34. Find the area of the region bounded by the ellipse $\frac{x^2}{16} + \frac{y^2}{9} = 1$

Solution:

Here $\frac{x^2}{16} + \frac{y^2}{9} = 1$ …(1)

It is a horizontal ellipse having center at origin and is symmetrical about both axes (if we change y to -y or x to -x, equation remain same). **$\frac{1}{2}$**

Standard equation of an ellipse is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

By comparing, a = 4 and b = 3

From equation (1)

$\Rightarrow y^2 = \frac{9}{16}(16 - x^2)$

$\Rightarrow y = \frac{3}{4}\sqrt{16 - x^2}$ …..(2)

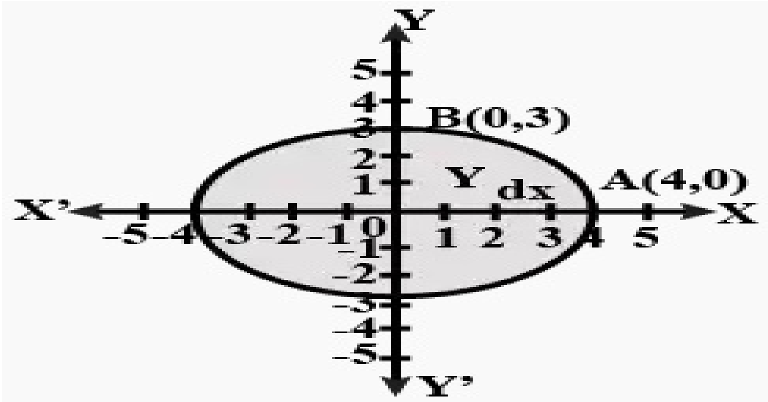
Points of Intersections of ellipse (1) with x-axis (y = 0)
Put y = 0 in equation (1), we have

$x^2/16 = 1$
$\Rightarrow x^2 = 16$
$\Rightarrow x = \pm 4$
Therefore, Intersections of ellipse(1) with x-axis are (0, 4) and (0, -4).

Now again, **1**
Points of Intersections of ellipse (1) with y-axis (x = 0)
Putting x = 0 in equation (1), $y^2/9 = 1$
$\Rightarrow y^2 = 9$
$\Rightarrow y = \pm 3$

Therefore, Intersections of ellipse (1) with y-axis are (0, 3) and (0,-3).

for arc of ellipse in first quadrant.



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Now,
Area of region bounded by ellipse (1)
Total shaded area = 4 x Area OAB of ellipse in first quadrant **$\frac{1}{2}$**

$=4\left|\int_0^4 y.\,dx\right|$ [∵ at end B of arc AB of ellipse: x=0 and at end A of arc AB; x=4]

$=4\left|\int_0^4 \frac{3}{4}\sqrt{16-x^2}.\,dx\right| = 3\left|\int_0^4 \sqrt{4^2-x^2}.\,dx\right|$ **1**

$=3\left|\frac{x}{2}\sqrt{4^2-x^2}+\frac{4^2}{2}\sin^{-1}\frac{x}{4}\right|_0^4$ $\left[\because \int\sqrt{a^2-x^2}\,dx = \frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$

$3[((4/2)\sqrt{16-16} + 8\sin^{-1}1) - (0+8\sin^{-1}0)] = 3[0+(8\pi/2)]$

$=3(4\pi) = 12\pi$ sq. units **1**

Question 35.

Solve the following problem graphically:

Minimise and Maximise $Z = 3x + 9y$

Subject to the constraints: $x + 3y \leq 60$

$x + y \geq 10$

$x \leq y$

$x \geq 0, y \geq 0$

Solution:

$Z = 3x + 9y.$ (1)
$x + 3y \leq 60$ (2)
$x + y \geq 10.$ (3)
$x \geq y$ (4)
$x \geq 0 , y \geq 0$ (5)

First of all, let us graph the feasible region of the system of linear inequalities (2) to (5).
Let $Z= 3x + 9y$ ....(1)
Converting inequalities to equalities

$x + 3y = 60$

X	0	60
Y	20	0

Points are (0, 20), (60,0)

Now put (0, 0) in inequation (2),

we find $0 \le 60$, which is true. ½
Therefore area lies towards the origin from this line.

$x + y = 10$

x	0	10
y	10	0

Points are (0, 10), (10, 0)

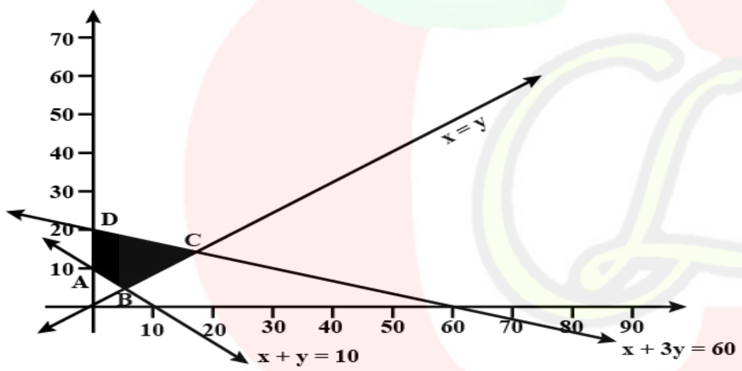
Now put (0, 0) in inequation (3),
we find $0 \ge 10$, which is False. ½
Therefore area lies away from the origin from this line.

$x - y = 0$

X	0	10	20
y	0	10	20

Points are (0,0),(10,10),(20,20)
Now put (1, 0) in inequation (4),
we find $1 \ge 0$, which is false. ½
Therefore area lies away from the (1, 0) from this line.

Plot the graph for the set of points



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To find maximum and minimum
The feasible region ABCD is shown in the figure. Note that the region is bounded. The coordinates of the corner points A, B, C and D are (0, 10), (5, 5), (15, 15) and (0, 20) respectively. ½

Corner Point	Corresponding Value of $Z = 3x + 9y$
A (0, 10)	90
B (5, 5)	**60←Minimum**
C (15, 15)	**180←Maximum**
D (0, 20)	**180 (**Multiple optimal solutions)

$1\frac{1}{2}$

We now find the minimum and maximum value of Z.
From the table, we find that the minimum value of Z is 60 at the point B (5, 5) of the feasible region.
The maximum value of Z on the feasible region occurs at the two corner points C (15, 15) and D (0, 20) and it is 180 in each case. ½

	SECTION – E (4Marks × 3Q)	
Question 36.	The proportion of a river's energy that can be obtained from an undershot water wheel is $E(x) = 2x^3 - 4x^2 + 2x$, units where x is the speed of the water wheel relative to the speed of the river. *Based on the above information answer the following :* (i) Find the maximum value of E(x) in the interval [0, 1]. (2) (ii) What is the speed of water wheel for maximum value of E(x)? (1) (iii) Does your answer agree with Mill wrights rule that the speed of wheel should be about one-third of the speed of the river? (1)	
Solution:	(i) We have, $E(x) = 2x^3 - 4x^2 + 2x$ …(1) Differentiating equation (1) w.r.t. x $E'(x) = 6x^2 - 8x + 2$ …(2) For maximum or minimum value of E(x), E'(x) = 0 we have $6x^2 - 8x + 2 = 0$ $3x^2 - 4x + 1 = 0$ $(3x - 1)(x - 1) = 0$ i.e. $x = 1/3$, $x = 1$ Differentiating equation (2) w.r.t. x $E''(x) = 12x - 8$ Now , At x= 1 $E''(x) = 12(1) - 8 = 4 = +ve$ At x = 1/3 $E''(x) = 12(1/3) - 8 = -4 = -ve$ ⇒ E(x) has maximum value at x = 1/3 Maximum value $= E(1/3) = 2(1/3)^3 - 4(1/3)^2 + 2(1/3)$ $= 2/27 - 4/9 + 2/3 = 8/27$	1 1
	(ii) Speed for the Maximum value of E(x) is $\frac{1}{3}$ units.	1
	(iii) Yes	1
Question 37.	A linear differential equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then such equation is known as linear differential equation. Its solution is given by $y.(IF.) = \int Q(IF.)\,dx + c$, where I.F.(Integrating Factor) $= e^{\int P dx}$ Now, suppose the given equation is $xdy + ydx = x^3\,dx$ *Based on the above information, answer the following questions:* (i)What are the values of P and Q respectively? (1) (ii) What is the value of I.F.? (1) (iii) Find the Solution of given equation. (2)	

Solution:	(i) Given equation is x.dy + y.dx = x^3 dx Dividing on both side by dx, we have $x\frac{dy}{dx} + y = x^3$ $\frac{dy}{dx} + \frac{1}{x}y = x^2$ $\Rightarrow P = \frac{1}{x}$, $Q = x^2$	1
	(ii) I.F.(Integrating Factor) = $e^{\int Pdx}$ $= e^{\int \frac{1}{x}dx}$ $= e^{\log x}$ $= x$	1
	(iii) Solution of given equation is y.(IF.) = $\int$ Q(IF.) dx +c $y(x) = \int x^2(x)\, dx + c$ $xy = \int x^3 dx + c$ $xy = \frac{x^4}{4} + c$	2
Question 38.	Ratna has two boxes I and II. Box I contains 3 red and 6 black balls. Box II contains 5 red and 5 black balls. Her friend Shivani selects one of the two boxes randomly and draws a ball out of it. The ball drawn by Shivani is found to be red. Let E_1 , E_2 and A denote the following events: E_1 : Box I is selected by Shiavni. E_2 : Box II is selected by Shiavni. A : Red ball is drawn by Shivani. (a) Find $P(E_1)$ and $P(E_2)$ (1) (b) Find $P(A\|E_1)$ and $P(A\| E_2)$ (1) (c) Find $P(E_2 \| A)$ (2)	
Solution:	(a) $P(E_1)$: Probability of selecting Box I by Shiavni = $\frac{1}{2}$ $P(E_1)$: Probability of selecting Box I by Shiavni = $\frac{1}{2}$	1
	(b) $P(A\|E_1)$ = Probability of selecting a red ball when box I has been already selected = $\frac{3}{9}$ $P(A\|E_2)$ = Probability of selecting a red ball when box II has been already selected = $\frac{5}{10}$	1
	(c) $P(E_2 \| A)$ = Probability that a red ball is drawn from the box II By Bayes' Theorem $P(E_2 \mid A) = \frac{P(E_2).P(A\mid E_2)}{P(E_1).P(A\mid E_1) + P(E_2).P(A\mid E_2)}$	

	$P(E_2 \mid A) = \frac{\frac{1}{2}.\frac{5}{10}}{\frac{1}{2}.\frac{3}{9}+\frac{1}{2}.\frac{5}{10}}$ $P(E_2 \mid A) = \frac{\frac{1}{4}}{\frac{1}{6}+\frac{1}{4}}$ $P(E_2 \mid A) = \frac{\frac{1}{4}}{\frac{4+6}{24}} = \frac{1}{4} \times \frac{24}{10} = \frac{3}{5}$	2