BSEH Practice Paper (March 2024)

(2023-24)

Marking Scheme

MATHEMATICS

SET-C

CODE: 835

⇨ Important Instructions: ● All answers provided in the Marking scheme are SUGGESTIVE
● Examiners are requested to accept all possible alternative correct answer(s).

	SECTION – A (1Mark × 20Q)	
Q. No.	**EXPECTED ANSWERS**	**Marks**
Question 1.	Let R be the relation in the set **N** given by R = {(a, b) : $\lvert a - b \rvert$ is a multiple of 4, b < 4}. Choose the correct answer.	
Solution:	**(C) (15, 3) ∈ R**	1
Question 2	$\sin^{-1}(\sin\frac{3\pi}{5})$ is equal to	
Solution:	**(B)** $\frac{2\pi}{5}$	1
Question 3	If A = $\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$, and A + A′ = I, then the value of α is	
	(B) $\frac{\pi}{3}$	1
Question 4.	If A is a square matrix of order 3×3 such that $\lvert A \rvert$ =4, then value of $\lvert 3A \rvert$ is	
Solution:	**(B) 108**	1
Question 5.	If the vertices of a triangle are (2, 7), (1, 1) and (10, 8), then by using determinants its area is	
Solution:	**(B)** $\frac{47}{2}$	1
Question 6.	If y = logx-x^2, then $\frac{d^2y}{dx^2}$ is equal to:	

Solution:	**(C)** $\frac{-1}{x^2} - 2$	1
Question 7.	If $\frac{d}{dx}$ f(x) = $4x^{3/2} - \frac{3}{x^4}$, then f(x) is	
Solution:	(D) $\frac{8}{5}x^{5/2} + \frac{1}{x^3} + C$	1
Question 8.	$\int e^x(\sin x + \cos x)\, dx$ equals:	
Solution:	(A) $e^x \sin x + C$	1
Question 9.	The value of $\int_0^1 \frac{1}{1+x^2} dx$ is	
Solution:	(D) $\frac{\pi}{4}$	1
Question10.	The order of the differential equation $\frac{d^2y}{dx^2} - 3(\frac{dy}{dx})^2 + y = 0$ is :	
Solution:	(A) 2	1
Question11.	Find the integrating factor of the differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$.	
Solution:	$e^{\int \frac{1}{x} dx} = e^{\log x} = x$	1
Question12.	If $x = 2at^2$, $y = at^4$ then find $\frac{dy}{dx}$.	
Solution:	$\frac{dx}{dt} = 4at$ and $\frac{dx}{dt} = 4at^3$ $\frac{dy}{dx} = \frac{dy/dt}{dx/dt} = \frac{4at^3}{4at} = t^2$	1
Question13.	Find the direction cosines of z-axis.	
Solution:	$< 0, 0, 1 >$	1
Question14.	If $P(A) = \frac{6}{11}$ $P(B) = \frac{5}{11}$ and $P(A \cup B) = \frac{7}{11}$, find P(A/B).	
Solution:	$P(A \cap B) = P(A) + P(B) - P(A \cup B)$ $P(A \cap B) = \frac{6}{11} + \frac{5}{11} - \frac{7}{11} = \frac{4}{11}$	1

	$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{4/11}{5/11} = \frac{4}{5}$	
Question15.	If $\vec{a}$ and $\vec{b}$ are two adjacent sides of a square then $\vec{a}.\vec{b} = 0$. (True / False)	
Solution:	True	1
Question16.	If A and B are independent events, then A' and B' are also independent. (True / False)	
Solution:	True	1
Question17.	The value of $\hat{\imath}.(\hat{\jmath} \times \hat{k}) + \hat{\jmath}.(\hat{\imath} \times \hat{k}) + \hat{k}.(\hat{\imath} \times \hat{\jmath})$ is …………………	
Solution:	1	1
Question18.	Two events E and F associated with a random experiment are ……..if the probability of occurrence or non occurrence of E is not affected by the occurrence or non occurrence of F.	
Solution:	Independent	
Question19.	**Assertion (A):** If R is the relation in set {1, 2, 3, 4, } given by R = { (1,2), (2,2), (1,1),(4,4),(1,3),(3,3),(3,2) } then R is not an equivalence relation. **Reason (R):** A relation is said to be an equivalence relation if it is reflexive, symmetric and transitive.	
Solution:	**(A)**	1
Question20.	**Assertion (A):**Three lines with direction cosines $<\frac{12}{13}, \frac{-3}{13}, \frac{-4}{13}>$; $<\frac{4}{13}, \frac{12}{13}, \frac{3}{13}>$ $<\frac{3}{13}, \frac{-4}{13}, \frac{12}{13}>$ are mutually perpendicular. **Reason (R):**Two lines with direction cosines l_1, m_1, n_1 and l_2, m_2, n_2 are perpendicular to each other if $l_1l_2 + m_1m_2 + n_1n_2 \neq 0$	
Solution:	(C)	1
	SECTION – B (2Marks × 5Q)	
Question21.	Check the injectivity and surjectivity of the function f**: Z→Z** given by f(x)= |x|	

Solution:	Here given function is $f(x)=\|x\|$ It is seen that $f(-1) = \|-1\| = 1$ and $f(1) = \|1\| = 1$ but $-1 \neq 1$. Hence **f** is not injective Now $-2 \in \mathbf{Z}$ but their does not exist any element $x\in \mathbf{Z}$ such that $f(x)= -2$ i.e. $\|x\| = -2$ hence **f** is not surjective Hence the function is neither injective nor surjective.	**1** 1
OR Question21.	Write in the simplest form to the function: $\tan^{-1}\left[\sqrt{\frac{1-\cos x}{1+\cos x}}\right]$, $0 < x < \pi$	
Solution:	$\tan^{-1}\left[\sqrt{\frac{1-\cos x}{1+\cos x}}\right] = \tan^{-1}\left[\sqrt{\frac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right]$ $0 < x < \pi$ $= \tan^{-1}\left[\tan\frac{x}{2}\right]$ $= \frac{x}{2}$	1 1
Question 22.	Find the value of X and Y if $X+Y=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ and $X-Y=\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$	
Solution:	$X + Y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$ ....(1) $X - Y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ ....(2) adding (1) and (2) $2X=\begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}+\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$	

	$2X=\begin{bmatrix}10 & 0\\ 2 & 8\end{bmatrix}$ $X=\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$ Putting the value of X in equation 1 $\begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix} + Y = \begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix}$ $Y = \begin{bmatrix}7 & 0\\ 2 & 5\end{bmatrix} - \begin{bmatrix}5 & 0\\ 1 & 4\end{bmatrix}$ $Y = \begin{bmatrix}2 & 0\\ 1 & 1\end{bmatrix}$	1 1
Question23.	Find the value of k so that the function is continuous is at x = 5. $f(x) = \begin{cases}\frac{x^2-25}{x-5}, & x \neq 5\\ k & x = 5\end{cases}$	
Solution:	Given function is $f(x) = \begin{cases}\frac{x^2-25}{x-5}, & x \neq 5\\ k & x = 5\end{cases}$ Now $\lim_{x\to 5} f(x) \Rightarrow \lim_{x\to 5} \frac{x^2-25}{x-5}$ $\lim_{x\to 5} \frac{(x-5)(x+5)}{x-5} = \lim_{x\to 5}(x+5) = 10$ Since function is continuous, therefore $\lim_{x\to 1} f(x) = f(1)$ k = 10	**1** **1**
Question24.	Verify that the function $y = e^x + 1$, is a solution of the differential equation $\frac{d^2y}{dx^2} - \frac{dy}{dx} = 0$	
Solution:		

	The given function is $y = e^x + 1$ $\frac{dy}{dx} = e^x$ $\frac{d^2y}{dx^2} = e^x$ L.H.S. $= \frac{d^2y}{dx^2} - \frac{dy}{dx}$ $= e^x - e^x = 0$ => L.H.S. = R.H.S.	$\frac{1}{2}$ $\frac{1}{2}$ 1
OR Question24.	Solve the differential equation $\frac{dy}{dx} = (1+ y^2)(1+ x^2)$.	
Solution:	The given equation is $\frac{dy}{dx} = (1+ y^2)(1+ x^2)$ ⇨ $\frac{dy}{(1+y^2)} = (1+ x^2).dx$ Integrating both sides , we have $\tan^{-1} y = x + \frac{x^2}{2} + C$ which is the required solution.	$\frac{1}{2}$ $1\frac{1}{2}$
Question25.	Two cards are drawn at random and without replacement from a pack of 52 cards find the probability that both the cards are black.	
Solution:	There are 26 black cards in a deck of 52 cards Let P(A)= the probability of getting a black card on the first draw Let P(B)= the probability of getting a black card on the second draw. Therefore, P(A) $= \frac{26}{52} = \frac{1}{2}$ Since the second card is not replaced P(B)= $\frac{25}{51}$ Thus probability of getting both the cards black=P(A).P(B)	1

	$=\frac{1}{2}\times\frac{25}{51}$ $=\frac{25}{102}$	**1**
	SECTION – C (3Marks × 8Q)	
Question26.	Show that the relation R in the set a of all the books in a library of a college given by R ={(x, y): x and y have same number of pages} is an equivalence relation.	
Solution:	Set A is the set of all the books in the library of a college. R ={(x, y): x and y have same number of pages} Now R is **reflexive** since $(x, x) \in R$ as x and x has the same number of pages. Let $(x, y) \in R$ $\Rightarrow$ x and y have the same number of pages $\Rightarrow$ y and x have the same number of pages $\Rightarrow (y, x) \in R$ Therefore R is **symmetric** Now let $(x, y) \in R$ and $(y, z) \in R$ $\Rightarrow$ x and y have the same number of pages and y and z have the same number of pages $\Rightarrow$ x and z have the same number of pages $\Rightarrow (x, z) \in R$ Therefore R is **transitive.** Hence **R** is an **equivalence relation**.	$\frac{1}{2}$ 1 1 $\frac{1}{2}$
OR Question26.	Write $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$, $a > 0$; $\frac{-a}{\sqrt{3}} \le x \le \frac{a}{\sqrt{3}}$ in the simplest form.	

Solution:	We have $\tan^{-1}\left(\frac{3a^2x-x^3}{a^3-3ax^2}\right)$ $a > 0;$ $\frac{-a}{\sqrt{3}} \leq x \leq \frac{a}{\sqrt{3}}$ Put $x = \tan\theta$, we have $\tan^{-1}\left(\frac{3a^2.a\tan\theta-a^3\tan^3\theta}{a^3-3a.a^2\tan^2\theta}\right)$ $=\tan^{-1}\left(\frac{3a^3\tan\theta-a^3\tan^3\theta}{a^3-3a^3\tan^2\theta}\right)$ $=\tan^{-1}\left(\frac{a^3(3\tan\theta-\tan^3\theta)}{a^3(1-3\tan^2\theta)}\right)$ $=\tan^{-1}\frac{(3\tan\theta-\tan^3\theta)}{(1-3\tan^2\theta)}$ $=\tan^{-1}\tan3\theta$ $= 3\theta$ $=3\tan^{-1}\frac{x}{a}$	$\frac{1}{2}$ 1 1 $\frac{1}{2}$
Question27.	Given A= $\begin{bmatrix}2 & 4 & 0\\3 & 9 & 6\end{bmatrix}$ and B = $\begin{bmatrix}1 & 4\\2 & 8\\1 & 3\end{bmatrix}$. Is $(AB)' = B'A'$?	
Solution:	$AB = \begin{bmatrix}2 & 4 & 0\\3 & 9 & 6\end{bmatrix}\begin{bmatrix}1 & 4\\2 & 8\\1 & 3\end{bmatrix}$ $=\begin{bmatrix}2+8+0 & 8+32+0\\3+18+6 & 12+72+18\end{bmatrix} = \begin{bmatrix}10 & 40\\27 & 102\end{bmatrix}$ $(AB)' = \begin{bmatrix}10 & 40\\27 & 102\end{bmatrix}'$ $(AB)' = \begin{bmatrix}10 & 27\\40 & 102\end{bmatrix}$ Now $B'A' = \begin{bmatrix}1 & 2 & 1\\4 & 8 & 3\end{bmatrix}\begin{bmatrix}2 & 3\\4 & 9\\0 & 6\end{bmatrix}$	1 $\frac{1}{2}$

	$= \begin{bmatrix} 2+8+0 & 3+18+6 \\ 8+32+0 & 12+72+18 \end{bmatrix}$ $= \begin{bmatrix} 10 & 27 \\ 40 & 102 \end{bmatrix}$ ⇨ $(AB)' = B'A'$	**1** $\mathbf{\frac{1}{2}}$
Question28.	If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2 y_2 + x y_1 + y = 0$	
Solution:	Given: $y = 3\cos(\log x) + 4\sin(\log x)$, $\frac{dy}{dx} = \frac{d}{dx}(3\cos(\log x) + 4\sin(\log x))$ $= -3\sin(\log x).\frac{1}{x} + 4\cos(\log x).\frac{1}{x}$ $x\frac{dy}{dx} = -3\sin(\log x) + 4\cos(\log x)$ $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{d}{dx}(-3\sin(\log x) + 4\cos(\log x))$ $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -3\cos(\log x).\frac{1}{x} - 4\sin(\log x).\frac{1}{x}$ $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{1}{x}(3\cos(\log x) + 4\sin(\log x))$ $x\frac{d^2y}{dx^2} + \frac{dy}{dx} = -\frac{1}{x}y$ $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y$ $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ $x^2 y_2 + x y_1 + y = 0$ Hence proved	1 1 1
Question29.	A stone is dropped into a quiet lake and waves move in circles at the speed of 5 cm/s. At the instant when the radius of the circular wave is 8	

	cm, how fast is the enclosed area increasing?	
Solution:	Let r cm be the radius of the circular wave at any instant. Therfore, $\frac{dr}{dt} = 5$ cm/s Now , the area of the circular wave is given as $A = \pi r^2$ Diff. w.r.t. 't' $\frac{dA}{dt} = 2\pi r$ cm/s^2 Instant when r = 8 cm $\frac{dA}{dt} = 2\pi(8)$ cm/s^2 $\frac{dA}{dt} = 16\pi$ cm/s^2	$\frac{1}{2}$ $\frac{1}{2}$ 1 1
Question 30	Integrate: $\int \frac{x}{(x+1)(x+2)}\,dx$	
Solution:	$I = \int \frac{x}{(x+1)(x+2)}\,dx$ $\frac{x}{(x+1)(x+2)} = \frac{A}{(x+1)} + \frac{B}{(x+2)}$ ⇨ $x = A(x+2) + B(x+1)$ Put $x = -1$, $-1 = A(-1+2) + B(-1+1) \Rightarrow -1 = A$ Put $x = -2$, $-1 = A(-2+2) + B(-2+1) \Rightarrow -1 = -B$ $\therefore$ $A = -1$ and $B = 1$ ⇨ $\frac{x}{(x+1)(x+2)} = \frac{-1}{(x+1)} + \frac{1}{(x+2)}$ ⇨ $I = \int \left(\frac{-1}{(x+1)} + \frac{1}{(x+2)}\right).dx$	$\frac{1}{2}$ 1

	$\Rightarrow$ I = $-$ log|x + 1| + log|x + 2| + C	$1\frac{1}{2}$
OR Question30.	Evaluate: $\int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$	
Solution:	$I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx \quad \ldots(1)$ Using property of definite integral $\int_0^a f(x)dx = \int_0^a f(a-x)dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin(\frac{\pi}{2}-x)}}{\sqrt{\sin(\frac{\pi}{2}-x)}+\sqrt{\cos(\frac{\pi}{2}-x)}}\,dx$ $I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\cos x}}{\sqrt{\cos x}+\sqrt{\sin x}}\,dx \quad \ldots(2)$ Adding (1) and (2) $2I = \int_0^{\frac{\pi}{2}} \frac{\sqrt{\sin x}+\sqrt{\cos x}}{\sqrt{\sin x}+\sqrt{\cos x}}\,dx$ $2I = \int_0^{\frac{\pi}{2}} 1\,dx$ $2I = \|x\|_0^{\frac{\pi}{2}}$ $2I = \frac{\pi}{2}$ $I = \frac{\pi}{4}$	1 1 1
Question31.	If $\vec{a}, \vec{b}, \vec{c}$ are unit vectors such that $\vec{a}+\vec{b}+\vec{c}=\vec{0}$, find the value of $\vec{a}.\vec{b}+\vec{b}.\vec{c}+\vec{c}.\vec{a}$.	
Solution:	Given that $\|\vec{a}\| = \|\vec{b}\| = \|\vec{c}\| = 1$ We know $\|\vec{a}\|^2 = \vec{a}.\vec{a}$	$\frac{1}{2}$

	$\therefore \left|\vec{a} + \vec{b} + \vec{c}\right|^2 = (\vec{a} + \vec{b} + \vec{c}).(\vec{a} + \vec{b} + \vec{c})$ Since $\vec{a} + \vec{b} + \vec{c} = \vec{0}$, therefore $\left|\vec{0}\right|^2 = \vec{a}.\vec{a} + \vec{a}.\vec{b} + \vec{a}.\vec{c} + \vec{b}.\vec{a} + \vec{b}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} + \vec{c}.\vec{b} + \vec{c}.\vec{c}$ $\left|\vec{0}\right|^2 = |\vec{a}|^2 + \left|\vec{b}\right|^2 + |\vec{c}|^2 + 2\vec{a}.\vec{b} + 2\vec{b}.\vec{c} + 2\vec{c}.\vec{a}$ $[\because \vec{a}.\vec{b} = \vec{b}.\vec{a}]$ $0 = 1 + 1 + 1 + 2\,(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a})$ $2\,(\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}) = -3$ $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a} = \frac{-3}{2}$	$\frac{1}{2}$ $1\frac{1}{2}$ $\frac{1}{2}$
	SECTION – C (5Marks × 4Q)	
Question32.	Solve the system of equations $x - y + z = 4$ $2x + y - 3z = 0$ $x + y + z = 2$	
Solution:	$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ $\lvert A\rvert = 1(1 + 3) + 1(2 + 3) + 1(2 - 1) = 1(4) + 1(5) + 1(1)$ $= 4 + 5 + 1$ $= 10 \neq 0$; Inverse of matrix exists. To find the inverse of matrix: Cofactors of matrix: $A_{11} = 4$, $A_{12} = -5$, $A_{13} = 1$ $A_{21} = 2$, $A_{22} = 0$, $A_{23} = -2$ $A_{31} = 2$, $A_{32} = 5$, $A_{33} = 3$	1

	$\Rightarrow$ adj.A = $\begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ So, $A^{-1} = \frac{\text{adj.A}}{\|A\|}$ $A^{-1} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ Now, matrix of equations can be written as: AX=B $\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ And, $X = A^{-1} B$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$ $\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$ Therefore, x = 2, y = -1 and z =1.	**1** **1** **1** **1**
Question33.	Find the area lying in the first quadrant and bounded by the circle $x^2 + y^2 = 4$ and the lines x = 0, x = 2.	

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<td>Solution:</td>
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Equation of the curve is $x^2 + y^2 = 4$

The area bounded by the circle $x^2 + y^2 = 4$……(1)

And the lines $x = 0$, $x = 2$ in the first quadrant is represented as.

Area of OAB = $\int_0^2 y\,dx$

From equation (1)

$\Rightarrow y^2 = (4 - x^2)$

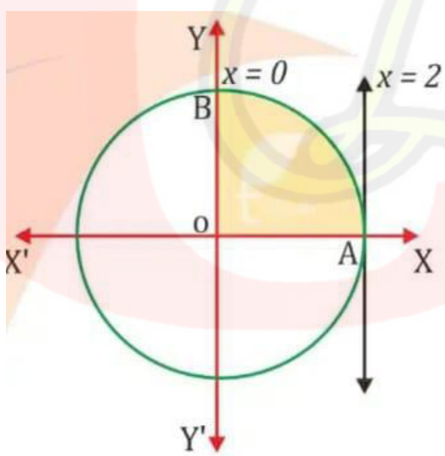
Points of intersections of given curves

At $x = 0$, $y = \sqrt{4} = \pm 2$ points are (0, 2) (0, -2)

At $x = 2$, $y = \sqrt{0} = 0$ points are (2, 0)

$\therefore$ points in first quadrant A(2, 0) B(0, 2)

Make a rough hand sketch of given curves by taking some corresponding values of x and y.



Required area is:

$\int_0^2 y\,dx = \int_0^2 \sqrt{4 - x^2}\,.dx$ [From equation (1)]

$= \left| \frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2} \right|_0^2$

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$1\frac{1}{2}$

1

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	$= [((2/2)\sqrt{4-4} + 2\sin^{-1}1) - 0(0 + 2\sin^{-1}0)]$ $= [0 + 2(\pi/2)]$ $= \pi$ sq units	$1\frac{1}{2}$ 1
OR Question33.	Find the area of the region bounded by the ellipse $\frac{x^2}{4} + \frac{y^2}{36} = 1$	
Solution:	Here $\frac{x^2}{4} + \frac{y^2}{36} = 1$ ...(1) It is a vertical ellipse having center at origin and is symmetrical about both axes (if we change y to -y or x to -x, equation remain same). Standard equation of an ellipse is $\frac{x^2}{b^2} + \frac{y^2}{a^2} = 1$ By comparing, $a = 6$ and $b = 2$ From equation (1) $\Rightarrow y^2 = \frac{36}{4}(4 - x^2) \Rightarrow y^2 = 9(4 - x^2)$ $\Rightarrow y = 3\sqrt{4 - x^2}$ .....(2) Points of Intersections of ellipse (1) with x-axis (y = 0) Put y = 0 in equation (1), we have $x^2/4 = 1$ $\Rightarrow x^2 = 4$ $\Rightarrow x = \pm 2$ Therefore, Intersections of ellipse(1) with x-axis are (2,0) and (-2, 0).	$\frac{1}{2}$ 1

Now again,

Points of Intersections of ellipse (1) with y-axis (x = 0)

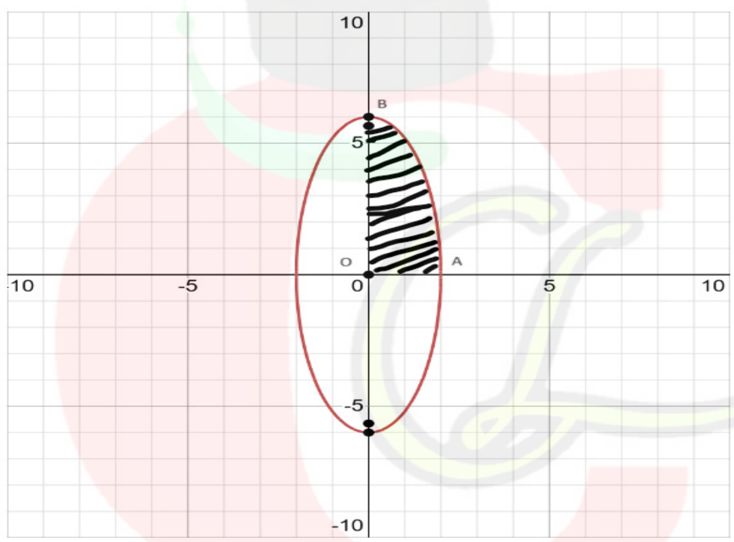
Putting x = 0 in equation (1), $y^2/36 = 1$

$\Rightarrow\ y^2 = 36$

$\Rightarrow\ y = \pm 6$

Therefore, Intersections of ellipse (1) with y-axis are (0, 6) and (0, −6)

for arc of ellipse in first quadrant.



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Now, Area of region bounded by ellipse (1)

Total shaded area = 4 x Area OAB of ellipse in first quadrant

$= 4\left|\int_0^2 y.\,dx\right|$ [∵ at end B of arc AB of ellipse: x = 0 and at end A of arc AB; x = 2] $\mathbf{\frac{1}{2}}$

$=4\left|\int_0^2 3\sqrt{4 - x^2}.\,dx\right| = 12\left|\int_0^2 \sqrt{2^2 - x^2}.\,dx\right|$

$= 12\left|\frac{x}{2}\sqrt{2^2 - x^2} + \frac{2^2}{2}\sin^{-1}\frac{x}{2}\right|_0^2$ $\left[\because \int\sqrt{a^2 - x^2}\,dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a}\right]$ **1**

	$12[((2/2)\sqrt{4-4}\ + 2\sin^{-1}1\ ) - (0 + 2\sin^{-1}0)] = 12[0 + (2\pi/2)]$ $= 12(\pi) = 12\pi$ sq. units	**1**
Question34.	Find the shortest distance between the lines l_1 and l_2 whose vector equations are $\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$ and $\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$	
Solution:	$\vec{r} = (1-t)\hat{\imath} + (t-2)\hat{\jmath} + (3-2t)\hat{k}$ $\vec{r} = (s+1)\hat{\imath} + (2s-1)\hat{\jmath} - (2s+1)\hat{k}$ $\vec{r} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k} + t(-\hat{\imath} + \hat{\jmath} - 2\hat{k})$ ……….(1) $\vec{r} = \hat{\imath} - \hat{\jmath} - \hat{k} + s(\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$ ……….(2) Comparing (1) and (2) with $\vec{r} = \overrightarrow{a_1} + \lambda\, \overrightarrow{b_1}$ and $\vec{r} = \overrightarrow{a_2} + \mu\, \overrightarrow{b_2}$ respectively, we get $\overrightarrow{a_1} = \hat{\imath} - 2\hat{\jmath} + 3\hat{k}$, and $\overrightarrow{b_1} = -\hat{\imath} + \hat{\jmath} - 2\hat{k}$ $\overrightarrow{a_2} = \hat{\imath} - \hat{\jmath} - \hat{k}$ and $\overrightarrow{b_2} = \hat{\imath} + 2\hat{\jmath} - 2\hat{k}$ Therefore $\overrightarrow{a_2} - \overrightarrow{a_1} = (\hat{\imath} - \hat{\jmath} - \hat{k}) - (\hat{\imath} - 2\hat{\jmath} + 3\hat{k})$ $= \hat{\jmath} - 4\hat{k}$ and $\overrightarrow{b_1} \times \overrightarrow{b_2} = (-\hat{\imath} + \hat{\jmath} - 2\hat{k}) \times (\hat{\imath} + 2\hat{\jmath} - 2\hat{k})$ $= \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ -1 & 1 & -2 \\ 1 & 2 & -2 \end{vmatrix} = \hat{\imath} - 4\hat{\jmath} - 3\hat{k}$ $\lvert \overrightarrow{b_1} \times \overrightarrow{b_2} \rvert = \sqrt{1 + 16 + 9} = \sqrt{26}$	1 $\frac{1}{2}$ $1\frac{1}{2}$

	Hence, the shortest distance between the given lines is given by D = $\frac{\|(\overrightarrow{a_2}-\overrightarrow{a_1}).(\overrightarrow{b_1}\times\overrightarrow{b_2})\|}{\|\overrightarrow{b_1}\times\overrightarrow{b_2}\|} = \frac{\|(\hat{j}-4\hat{k}).(\hat{\imath}-4\hat{j}-3\hat{k})\|}{\sqrt{26}} = \frac{\|-4+12\|}{\sqrt{26}} =$ D = $\frac{8}{\sqrt{26}}$ $= \frac{8}{\sqrt{26}} \times \frac{\sqrt{26}}{\sqrt{26}}$ $= \frac{8\sqrt{26}}{26} = \frac{4\sqrt{26}}{13}$ units Therfore the shortest distance between two lines is $\frac{3\sqrt{2}}{2}$ units	$1\frac{1}{2}$ $\frac{1}{2}$
OR Question34.	Find the vector equation of the line passing through the point (2,−1,3) and perpendicular to the two lines : $\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{2}$.	
Solution:	The vector equation of a line passing through a point with position vector $\vec{a}$ and parallel to $\vec{b}$ is $\vec{r} = \vec{a} + \lambda\,\vec{b}$ .It is given that, the line passes through (2, −1, 3). So, $\vec{a} = 2\hat{\imath} - \hat{j} + 3\hat{k}$ Given lines are $\frac{x-1}{2} = \frac{y-1}{-2} = \frac{z+1}{1}$ and $\frac{x-2}{1} = \frac{y+1}{2} = \frac{z-3}{2}$ It is also given that, line is perpendicular to both given lines. So we can say that the required line is perpendicular to both parallel vectors of two given lines. We know that, $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ & $\vec{b}$, so let $\vec{b}$ is cross product of parallel vectors of both lines i.e. $\vec{b} = \overrightarrow{b_1} \times \overrightarrow{b_2}$ where $\overrightarrow{b_1} = 2\hat{\imath} - 2\hat{j} + \hat{k}$ and $\overrightarrow{b_2} = \hat{\imath} + 2\hat{j} + 2\hat{k}$ and Required Normal	1 2

	$\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & -2 & 1 \\ 1 & 2 & 2 \end{vmatrix}$ $= \hat{\imath}(-4 - 2) - \hat{\jmath}(4 - 1) + \hat{k}(4 + 2)$ $\vec{b} = -6\hat{\imath} - 3\hat{\jmath} + 6\hat{k}$ Now, by substituting the value of $\vec{a}$ & $\vec{b}$ in the formula $\vec{r} = \vec{a} + \lambda\,\vec{b}$, we get $\vec{r} = (2\hat{\imath} - \hat{\jmath} + 3\hat{k}) + \lambda(-6\hat{\imath} - 3\hat{\jmath} + 6\hat{k})$	1 1
Question35.	Solve the following problem graphically: Minimise and Maximise Z = x + 2y Subject to the constraints: $x + 2y \geq 100$ $2x - y \leq 0$ $2x + y \leq 200$ $x, y \geq 0$	
Solution:	$Z = x + 2y$ ...(1) $x + 2y \geq 100$ ...(2) $2x - y \leq 0$ ...(3) $2x + y \leq 200$ ...(4) $x \geq 0, y \geq 0$ ...(5) First of all, let us graph the feasible region of the system of linear inequalities (2) to (5). Let Z= 200 x + 500 y ....(1) Converting inequalities to equalities $x + 2y = 100$ <table><tr><td>X</td><td>0</td><td>100</td></tr><tr><td>Y</td><td>50</td><td>0</td></tr></table>Points are (0, 50), (100, 0) Now Put (0, 0) in (2) inequation we have $0 \geq 100$ which is false. ∴ Req. region lies away from the origin. $2x - y = 0$	$\frac{1}{2}$

X	50	100
Y	100	200

Points are (50, 100), (100, 200)

Now Put (10, 10) in (3) inequation we have , $10 \leq 0$ which is false. $\frac{1}{2}$

$\therefore$ Req. region lies away from the point (10, 10).

$2x + y = 200$

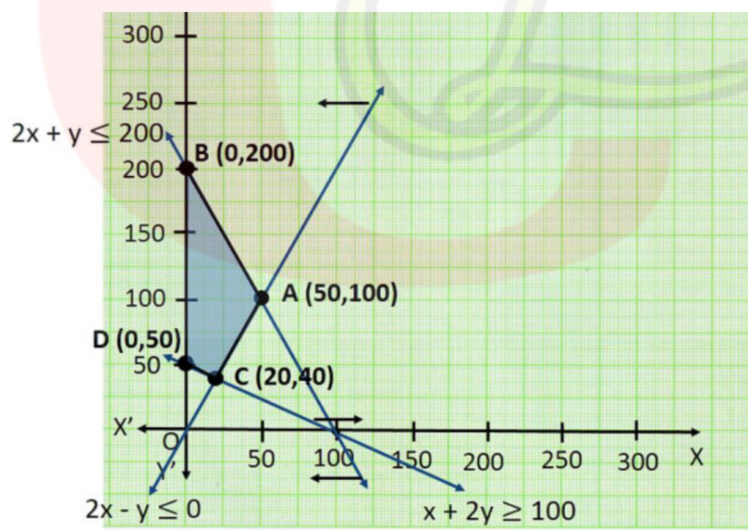
X	0	100
Y	200	0

Points are (0, 50), (100,0) $\frac{1}{2}$

Now Put (0, 0) in (2) inequation we have, $0 \leq 200$ which is true.

$\therefore$ Req. region lies towards the origin.

Plot the graph for the set of points



1

To find minimum

The feasible region ABCD is shown in the figure. Note that the region is bounded. The coordinates of the corner points A, B, C and D are (50, 100), (0, 200), (20, 40) and (0, 50) respectively. $\frac{1}{2}$

	<table><tr><td>Corner Point</td><td>Corresponding Value of Z = x + 2 y</td></tr><tr><td>A (50, 100) B (0, 200) C (20, 40) D (0, 50)</td><td>250 **400←Maximum** **100←Minimum** **100←Minimum**</td></tr></table> From the table, we find that, ∴The maximum value of Z is 400 at the point B (0, 200). The minimum value of Z is 100 at the point C (20, 40) and D (0, 50).	$1\frac{1}{2}$ $\frac{1}{2}$
	SECTION – E (4Marks × 3Q)	
Question36.	A square piece of tin of side 24 cm is to be made into a box without top by cutting a square of side x cm from each corner and folding up the flaps to form a box. On the basis of above information, answer the following questions (i) Write the length, breadth and height of the box formed in terms of x. (1) (ii) Express volume V of the box in terms of x. (1) (iii) Show that volume of the box is maximum, when x = 4 cm. (2)	
Solution:	Let x be the side of the square to be cut off from each of the corners. ∴ Length of the box formed = (24 – 2x) cm Breadth of the box formed = (24 – 2x) cm Height of the box formed = x cm	1

	$\therefore$ Volume of the box (V) = length × breadth × height $V = (24 - 2x) \times (24 - 2x) \times x$ $V = 4x^3 - 96x^2 + 576x$	1
	Volume of the box $= V = 4x^3 - 96x^2 + 576x$ …(1) Diff. w.r.t. 'x' $\frac{dV}{dx} = 12x^2 - 192x + 576$ $= 12(x^2 - 16x + 48)$ …(2) For miaximum or minimum value of V, $\frac{dV}{dx} = 0$ we have ⇨ $x^2 - 16x + 48 = 0$ ⇨ $(x - 12)(x - 4) = 0$ ⇨ $x = 12$ or $x = 4$ [Rejecting x = 12 as it is not possible] Diff. equation (2) again w.r.t. 'x', we have $\frac{d^2V}{dx^2} = 12(2x - 16)$ At x = 4 $\frac{d^2V}{dx^2} = 12(2(4) - 16) = -ve$ $\therefore$ Volume V is maximum at x = 4cm	2
Question37.	A linear differential equation is of the form $\frac{dy}{dx} + Py = Q$, where P, Q are functions of x, then such equation is known as linear differential equation. Its solution is given by $y.(IF.) = \int Q(IF.)\, dx + c$, where I.F.(Integrating Factor) $= e^{\int P dx}$ Now, consider the given equation is $(1 + \sin x)\frac{dy}{dx} + y\cos x = -x$ Based on the above information, answer the following questions: (i) What are the values of P and Q respectively? (1) (ii) What is the value of I.F.? (1)	

	(iii)Find the Solution of given equation. (2)	
Solution:	(i) Given differential equation is $(1+\sin x)\frac{dy}{dx}+y\cos x=-x$ Dividing on both side by $(1+\sin x)$, we have $$\frac{dy}{dx}+\frac{\cos x}{(1+\sin x)}y=\frac{-x}{(1+\sin x)}$$ Comparing this differential equation with $\frac{dy}{dx}+Py=Q$, we have $\Rightarrow P=\frac{\cos x}{(1+\sin x)}$ and $Q=\frac{-x}{(1+\sin x)}$	1
	(ii) I.F.(Integrating Factor) $=e^{\int Pdx}$ $=e^{\int\frac{\cos x}{(1+\sin x)}dx}$ $=e^{\log(1+\sin x)}$ I.F. $=1+\sin x$	1
	(iii) Solution of given equation is $y.(IF.)=\int Q(IF.)\,dx+c$ $y(1+\sin x)=\int\frac{-x}{(1+\sin x)}(1+\sin x)+c$ $y(1+\sin x)=-\int x\,dx+c$ $y(1+\sin x)=\frac{-x^2}{2}+c$	2
Question 38.	A doctor is to visit a patient. From the past experience, it is known that the probabilities that he will come by train, bus, scooter or by other means of transport are respectively $\frac{3}{10}$, $\frac{1}{5}$, $\frac{1}{10}$ and $\frac{2}{5}$ .The probabilities	

	that he will be late are $\frac{1}{4}$, $\frac{1}{3}$ and $\frac{1}{12}$ if he comes by train, bus and scooter respectively, but if he comes by other means of transport, then he will not be late. On the basis of above information, answer the following questions. (i) Find the probability that he is late. (ii) When he arrives, he is late. What is the probability that he comes by train? (iii) When he arrives, he is late. What is the probability that he comes by bus?	
Solution:	Let E_1, E_2, E_3 and E_4 be the events that the doctor comes by train, bus, scooter and other means of transport respectively. It is given that $P(E_1) = \frac{3}{10}$, $P(E_2) = \frac{1}{5}$, $P(E_3) = \frac{1}{10}$ and $P(E_4) = \frac{2}{5}$ Let A be the evnent that doctor visit the patient late . It is given that $P(A/E_1) = \frac{1}{4}$, $P(A/E_2) = \frac{1}{3}$, $P(A/E_3) = \frac{1}{12}$ and $P(A/E_4) = 0$	
	(i) Required probability that doctor is late = P(A) Therefore, $P(A) = P(E_1)P(A/E_1) + P(E_2)P(A/E_2) + P(E_3)P(A/E_3) + P(E_4)P(A/E_4)$ $P(A) = (\frac{3}{10})(\frac{1}{4}) + (\frac{1}{5})(\frac{1}{3}) + (\frac{1}{10})(\frac{1}{12}) + (\frac{2}{5})(0)$ $P(A) = \frac{3}{40} + \frac{1}{15} + \frac{1}{120}$ $P(A) = \frac{9+8+1}{120}$ $P(A) = \frac{18}{120}$ $P(A) = \frac{3}{20}$	2

	(ii) Probability that he comes by train given that he is late = $P(E_1/A)$ $\therefore\ P(E_1/A) = \dfrac{P(E1)P(A/E1)}{P(A)}$ $= \dfrac{(\frac{3}{10})(\frac{1}{4})}{\frac{3}{20}}$ $P(E_1/A) = \dfrac{3}{40} \times \dfrac{20}{3} = \dfrac{1}{2}$	1
	(iii) Probability that he comes by bus given that he is late = $P(E_2/A)$ $\therefore\ P(E_2/A) = \dfrac{P(E2)P(A/E2)}{P(A)}$ $= \dfrac{(\frac{1}{5})(\frac{1}{3})}{\frac{3}{20}}$ $P(E_2/A) = \dfrac{1}{15} \times \dfrac{20}{3} = \dfrac{4}{9}$	1